

Resultant Intensity Interference of two waves

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According to the given fig, suppose y_1 and y_2 two waves of vertical displacements that superimpose at a point P in space, the resultant displacement is represented as

$$y = y_1 + y_2$$

At the same time, these waves meet at some point P, and the only difference occurs in their phases.

$$y_1 = a \sin \omega t$$

$$y_2 = b \sin (\omega t + \phi)$$

where, a and b are their respective amplitudes. ϕ is the constant phase difference between the two waves.

On applying superposition principle, we get

$$y = a \sin \omega t + b \sin (\omega t + \phi) \quad \text{--- (1)}$$

Equation (1) is represented as the phasor diagram the resultant having amplitude A and a phase angle with respect to y_1 is given as

$$y = A \sin (\omega t + \theta)$$

$$A \sin (\omega t + \theta) = a \sin \omega t + b \sin (\omega t + \phi)$$

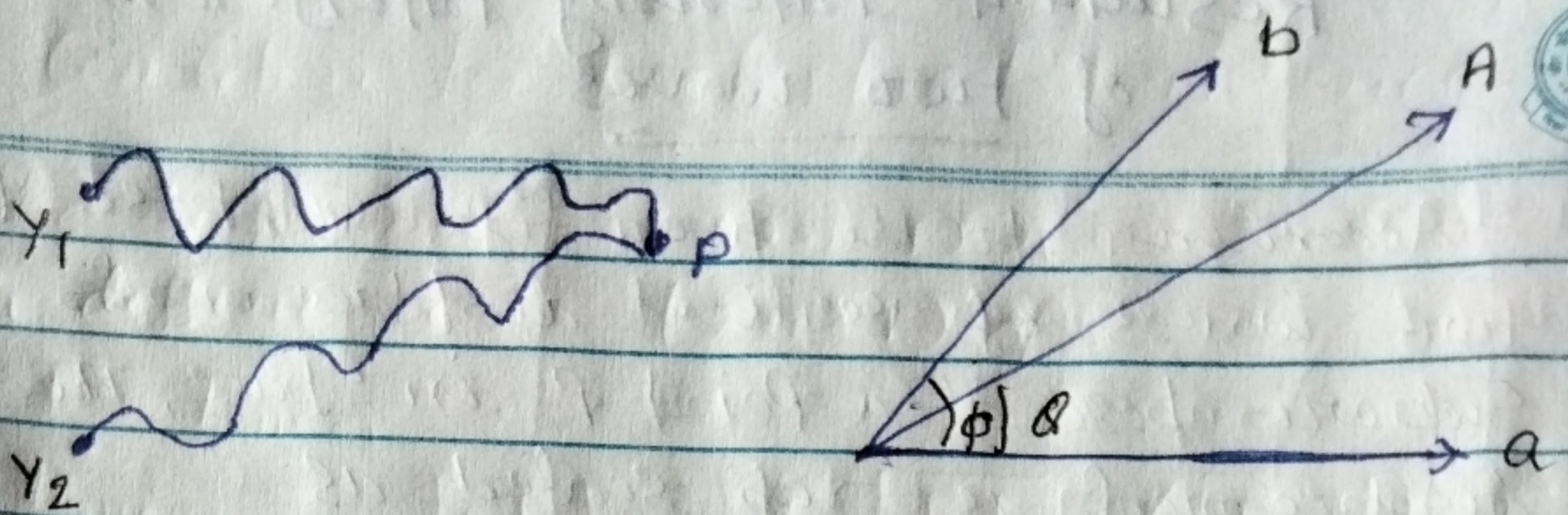
$$\text{Using } [\sin (A+B) = \sin A \cos B + \cos A \sin B]$$

$$A [\sin \omega t \cos \theta + \cos \omega t \sin \theta] = a \sin \omega t + b [\sin \omega t \cos \phi + \cos \omega t \sin \phi]$$

By equating 'sin ωt ' and 'cos ωt ' terms on both sides, we get

$$A \cos \theta = a + b \cos \phi \quad \text{--- (2)}$$

$$A \sin \theta = b \sin \phi \quad \text{--- (3)}$$



On Squaring and adding equⁿ (2) and equⁿ (3), we get

$$A^2 = (a + b \cos \theta)^2 + (b \sin \theta)^2$$

$$A^2 = a^2 + b^2 + 2ab \cos \theta$$

$$A = \sqrt{a^2 + b^2 + 2ab \cos \theta} \quad \text{--- (4)}$$

If we divide equⁿ (2) and equⁿ (3), we get

$$\tan \theta = \frac{b \sin \theta}{a + b \cos \theta} \quad \text{--- (5)}$$

Since intensity is indirectly proportional to the square of the amplitude of the wave

$$I \propto a^2$$

$$I_1 = ka^2, \quad I_2 = kb^2, \quad I = KA^2$$

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Taking

$$I_1 = KA^2$$

$$I = K(a^2 + b^2 + 2ab \cos \theta)$$

$$I = Ka^2 + Kb^2 + 2Kab \cos \theta$$

$$I = Ka^2 + Kb^2 + Kab \cos \theta$$

$$I = I_1 + I_2 + 2(\sqrt{Ka})(\sqrt{Kb}) \cos \theta$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \theta \quad \text{--- (6)}$$



When $\cos \theta = \pm 1$ then intensity should be maximum
($I = I_{\max}$) for constructive interference.

$$\theta = 0, 2\pi, 4\pi, \dots, (2n\pi)$$

If the path difference between the waves at point P is Δx , we have

$$\Delta x = \frac{\lambda}{2\pi} (\theta) ; \quad \Delta x = \frac{\lambda}{2\pi} (2n\pi)$$

$$n = 1, 2, 3, \dots$$

$$\Delta x = n\lambda$$

Condition for Constructive Interference:

Phase difference = 0 or $2n\pi$ where $n = 1, 2, 3, \dots$

Path difference = $n\lambda$ where $n = 1, 2, 3, \dots$

During constructive interference $I = I_{\max}$, then

$$I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = (\sqrt{I_1} + \sqrt{I_2})^2$$

$$= I_{\max} = K(a+b)^2$$

When $\cos \phi = -1$, intensity should be minimum
($I = I_{\min}$) for destructive interference.

$$\phi = \pi, 3\pi, 5\pi, \dots$$

$$\Rightarrow \phi = (2n-1)\pi, \text{ where } n = 1, 2, 3, \dots$$

of Δx is the path difference between the waves at point P.

$$\Delta x = \frac{\lambda}{2\pi} \phi ; \quad \Delta x = \frac{\lambda}{2\pi} (2n-1)\pi$$

Thus, we have $\Delta x = \frac{(2n-1)\lambda}{2}$

Condition for Destructive Interference

$$\text{Phase difference} = (2n-1)\pi$$

$$\text{Path difference} = (2n-1)\lambda/2$$

$$I = I_{\min}$$

$$I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2} = (\sqrt{I_1} - \sqrt{I_2})^2$$

Ex The two harmonic waves $y_1(x,t) = 0.2 \sin(x-3t)$ and $y_2(x,t) = 0.2 \sin(x-3t + \phi)$ are given. If $\phi = \pi/2$ rad then determine the expression of y . If ϕ between the waves is unknown and the amplitude of their sum is 0.32 m, then determine the value of ϕ .

Sol. Given that $y_1(x,t) = 0.2 \sin(x-3t)$

$$y_2(x,t) = 0.2 \sin(x-3t + \phi)$$

Since $y = y_1 + y_2$ then we have

$$= 0.2 \sin(x-3t) + 0.2 \sin(x-3t + \phi)$$

$$= 0.2 \sin(x-3t) + 0.2 \sin\left(x-3t + \frac{\pi}{2}\right)$$

$$= A \sin(x-3t + \theta)$$

$$A = \sqrt{(0.2)^2 + (0.2)^2} = 0.283 \text{ m}$$

$$\theta = \tan^{-1} \frac{0.2}{0.2} = \pi/4$$

Thus the expression of y is

$$y = 0.28 \sin\left(x-3t + \frac{\pi}{4}\right)$$

If amplitude of resulting wave is 0.32 m and $A = 0.2$ m, then

$$0.32 = \sqrt{(0.2)^2 + (0.2)^2} + 2(0.2)(0.2) \cos \phi$$

$$\Rightarrow \phi = 1.29 \text{ rad}$$

